Artificial Intelligence

Knowledge Representation I

Lecture 6

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Tralvex (Rex) Yeap
University of Leeds

Content: Knowledge Representation I

- Issues in Knowledge Representation
- Why logic in knowledge representation?
- Logical Symbols and Quantifiers
- How to use logical notation for representing knowledge
- A Set-Theoritical definition of L_{PC}
- Class Activity 1: Workouts on L_{PC}
- Ways of reasoning in L_{PC}
- Natural Deduction
- Semantic Tableau
- Truth Tables
- Relationships between (1) Natural Deduction, (2) Semantic Tableau, and (3) Truth Table (Model Theory) Methods
- What's in Store for Lecture 7

Quick Review on Lecture 5

- Class Activity 1: A* Search Workout for Romania Map
- Why Study Games?
- Game Playing as Search
- Special Characteristics of Game Playing Search
- Ingredients of 2-Person Games
- Normal and Game Search Problem
- A Game Tree for Naughts and Crosses
- A Perfect Decision Strategy Based on Minimaxing
- The Need for Imperfect Decision
- Assigning Utilities: Evaluation Functions
- When to Cut Search
- Chess Positions
- Alpha - Beta Pruning
- State of the Art in Checkers, Backgammon, Go & Chess
- Students' Mini Research Presentation by Group B
- Class Activity 2: Real-world Paper Reading & Practical - "The end of an era, the begin-ning of another? HAL, Deep Blue and Kasparov"
- Class Activity 3: Video on Deep Blue vs Kasparov (May, 1997) Game #6.
- Class Activity 4: Real-world Paper Reading & Practical - "Smart Moves - Intelligent Pathfinding"

Issues in Knowledge Representation

1. How to **represent** knowledge
2. How to **manipulate/process** knowledge

(2) Can be rephrased as: how to **make decisions** based on some knowledge.

Why logic in knowledge representation?

- Logical notation can be used to **express** info/knowledge.
- Logical notation is useful in **reasoning** about knowledge.

Why logic in knowledge representation?

1. Logical notation for expressing info/knowledge

AltaVista search: Mary AND lamb the output is:

$$\{p \in \text{WebPages} | \text{contain}(p,\text{Mary}) \ \& \ \text{contain}(p,\text{lamb})\}$$
$$= \{p \in \text{WebPages} | \text{contain}(p,\text{Mary}))\} \cap$$
$$\{p \in \text{WebPages} | \text{contain}(p,\text{lamp})\}$$

Why logic in knowledge representation?

2. Logical notation is useful in reasoning (about knowledge.

eg1. John is a human

if John is a human then John is mortal

therefore

John is mortal.

in logic: P

$P \rightarrow Q$

therefore

Q.

$$\frac{\begin{matrix} P \\ P \rightarrow Q \end{matrix}}{Q}$$

$$\frac{P \quad P \rightarrow Q}{Q}$$

Why logic in knowledge representation?

2. Logical notation is useful in reasoning (about knowledge)

eg1. John is a human

every human are mortals

therefore

John is mortal.

In logic:

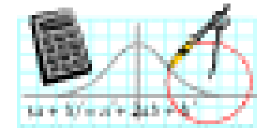
human(John)

$\forall h(\text{human}(h) \rightarrow \text{mortal}(h))$

therefore: $\text{human}(\text{John}) \rightarrow \text{mortal}(\text{John})$ by $\forall$ elim. rule

therefore: mortal(John) by $\rightarrow$elim. rule

Logical Symbols and Quantifiers



- Logical Symbols
  - ~ NOT
  - ∧ AND
  - ∨ OR
  - → IMPLIES
- Quantifers
  - ∀ FOR ALL
  - ∃ THERE EXISTS

Objective of this lecture

- Learn about logical symbols (and their formal meaning).
- Learn about the rules in using the symbols.
- Learn about expressing real life problems in terms of logical symbols.
- and probably more.

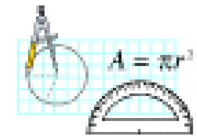
How to use logical notation for representing knowledge

- A particular logic, called: **Propositional Calculus**.
- Studying a logic --- studying a language
- Defn: A language of PC, call it L_{PC} is defined by the following rules:

  1. Variables p, q, r,p_2 ,q_2 ,r_2, ... are in L_{PC}. We call the above variables: undeterminate statements.
  2. If a statement A is in L_{PC} and a statement B is in L_{PC} , then the statement *(A&B)* is in L_{PC} .Similarly for the symbols: $\vee$, $\rightarrow$.
  3. If a statement A is in L_{PC}, then the statement ~A is in L_{PC} .

Note: L_{PC} is a set. It is a set of statements which can be generated from the above rules.

A Set-Theoritical definition of L_{PC}



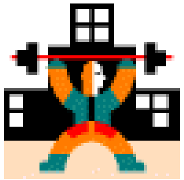
- **Mathematically**, we define L_{PC} as follows:

Rule 1: $p, q, r, p_2, q_2, r_2, \ldots \in L_{PC}$

Rule 2: If $A \in L_{PC}$ and $B \in L_{PC}$, then $(A \& B) \in L_{PC}$.
Similarly for the symbols: $\vee$, $\rightarrow$.

Rule 3: If $A \in L_{PC}$, then the statement $\sim A \in L_{PC}$.

Class Activity 1: Workouts on L_{PC}



- **Proof** whether or not the following are in L_{PC},

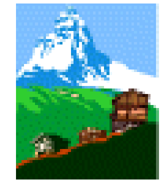
  1. $(A\rightarrow \sim B)$
  2. $(A\rightarrow B \sim)$
  3. $(A\sim\rightarrow B)$
  4. $(\sim A\rightarrow B)$
  5. $(((A\rightarrow B)\&(A\vee B)\rightarrow B)$

Ways of reasoning in L_{PC}

- 1. Natural deduction
- 2. Semantic tableau
- 3. Truth table (model theory)

Ways of reasoning in L_{PC}

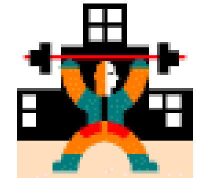
(1) Natural Deduction



Elimination Rules	Introduction Rules	
$\frac{A \wedge B}{A}$ $\quad$ $\frac{A \wedge B}{B}$	$\frac{A \quad B}{A \wedge B}$	$\wedge$
$\frac{A \vee B \quad \begin{matrix}[A] \\ \vdots \\ C\end{matrix} \quad \begin{matrix}[B] \\ \vdots \\ C\end{matrix}}{C}$	$\frac{A}{A \vee B}$ $\quad$ $\frac{B}{A \vee B}$	$\vee$
$\frac{A \to B \quad A}{B}$	$\frac{\begin{matrix}[A] \\ \vdots \\ B\end{matrix}}{A \to B}$	$\to$
$\frac{\begin{matrix}A \\ \vdots \\ \vdots \\ C\end{matrix} \quad \begin{matrix}A \\ \vdots \\ \vdots \\ {\sim}C\end{matrix}}{\otimes}$ $\quad$ $\frac{\otimes}{A}$	$\frac{\begin{matrix}A \\ \vdots \\ \vdots \\ \otimes\end{matrix}}{{\sim}A}$ $\quad$ $\frac{\begin{matrix}A \\ \vdots \\ \vdots \\ {\sim}A\end{matrix}}{\otimes}$	$\otimes$

$\otimes$ contradiction

Class Activity 2:
Workouts on Natural Deduction

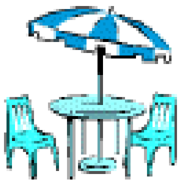


- **Resolve** the following using Natural Deduction:

1. $A \wedge B \rightarrow A \vee B$
2. $((A \wedge B) \wedge (A \rightarrow B)) \rightarrow B$
3. $A \vee \sim A$
4. $((A \rightarrow B) \rightarrow A) \rightarrow A$
5. $A \rightarrow \sim\sim A$
6. $\sim\sim A \rightarrow A$

Ways of reasoning in L_{PC}

(2) Semantic Tableau



- **Idea**: Similar to Natural Deduction

- Based on **Proof by Contradiction**

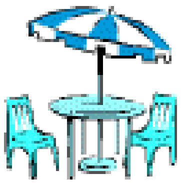
$$\frac{\begin{array}{c} \sim A \\ \vdots \\ \vdots \\ \otimes \end{array}}{A}$$

Rules in Semantic Tableau

A ∧ B | A | B	**A ∨ B** | ------------- | | A B		
A → B ≡ ~A ∨ B **~(A ∧ B)** ≡ ~A ∨ ~B **~(A ∨ B)** ≡ ~A ∧ ~B	**A → B** ≡ ~A ∨ B | ------------- | | ~A B	**~(A ∧ B)** ≡ ~A ∨ ~B | ------------- | | ~A ~B	**~(A ∨ B)** ≡ ~A ∧ ~B | ~A | ~B
A Closed Branch A / B / / ~B			

Rules in Semantic Tableau

Method



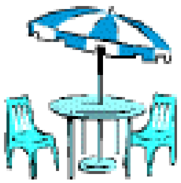
- Given Assumption: A_1, A_2, …,A_n , want to Deduce: B

  1. Start from $\mathbf{A_1}$: **Create tree** by applying the rules
  2. Mark any **closed Branch**
  3. **Repeat** (1-2) for $\mathbf{A_2}$**, …,** $\mathbf{A_n}$**, ~B**

  The proof is **completed** when:
  * We have applied $\mathbf{A_1}$**, …,** $\mathbf{A_n}$**, ~B**
  * All the branches are **closed**

Semantic Tableau Method

An Example



If we wish to check the validity of the following argument in English:

> Fred is ill or drunk.
> If he is drunk, he shouldn't drive.
> If he is ill, he shouldn't drive.
> So, Fred shouldn't drive.

the first step is to turn it into symbols by assigning propositional variables to appropriate 'atomic' propositions:

> A = 'Fred is ill.'
> B = 'Fred is drunk.'
> C = 'Fred shouldn't drive.'

and then use these to encode the argument in PC:

$$\frac{\begin{array}{c} A \vee B \\ A \rightarrow C \\ B \rightarrow C \end{array}}{C}$$

To check the validity of this argument using PC semantic tableau, we just check the consistency of the premises with the negation of the conclusion:

$$\begin{array}{c} A \vee B \\ A \rightarrow C \\ B \rightarrow C \\ \sim C \end{array}$$

If that set of sentences is inconsistent, then the conclusion is a theorem of the premises.

Applying the semantic tableau rules results in the tableau given in Figure 4.1. As all of the branches are closed, the argument is valid.

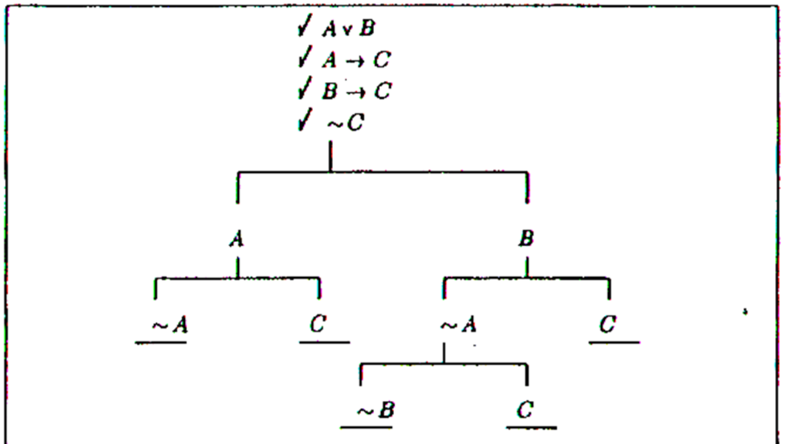


Figure 4.1: An example of a semantic tableau.

Class workout: **To show** that $((A \rightarrow B) \rightarrow A) \rightarrow A$ is true.

Ways of reasoning in L_{PC}

(3) Truth Tables

- A sentence *A* is **valid** means:

  1 occurs in all the rows of
  A's Truth Table

  equivalently means

  All possible interpretation
  of the sentence A gives out 1.

- An **interpretation**:

  A function from 'undeterminate variables' to {0, 1}

  eg. { p $\mapsto$ 0,
  q $\mapsto$ 0,
  r $\mapsto$ 1}

- Given an interpretation *I* and a sentence *S*, how do we interpret *S* with respect to *I*? Ans:

A Truth Assignment Function *I*

(1) For any atom p, either $I(p) = 1$ or $I(p) = 0$.
(2) For a well formed formula (wff) A, $I(\sim A) = 1$ if and only if (iff) $I(A) = 0$.
(3) For wffs A, B, $I(A \& B) = 1$ iff $I(A) = 1$ and $I(B) = 1$.
(4) For wffs A, B, $I(A \vee B) = 1$ iff $I(A) = 1$ or $I(B) = 1$.
(5) For wffs A, B, $I(A \rightarrow B) = 1$ iff $I(A) = 1$ whenever $I(A) = 1$.

Truth Tables

If we tabulate all of the possible values of the truth assignment function for the basic propositions and operations on those propositions, then we end up with ***truth tables***, as follows:

A	$\sim A$
0	1
1	0

A	B	$A \& B$
0	0	0
0	1	0
1	0	0
1	1	1

A	B	$A \vee B$
0	0	0
0	1	1
1	0	1
1	1	1

A	B	$A \rightarrow B$
0	0	1
0	1	1
1	0	0
1	1	1

Relationships between **(1)** Natural Deduction, **(2)** Semantic Tableau, and **(3)** Truth Table (Model Theory) Methods

- They have the same power,

  ie.

  S **provable** in Natural Deduction

  iff

  S **provable** in Semantic Tableau

  iff

  S is valid

- Provable $\xrightarrow{\text{sound}}$ Valid, Valid $\xrightarrow{\text{complete}}$ Provable

What’s in Store for Lecture 7

- Knowledge Representation II

End of Lecture 6

Good Night.